Handout for 2020-02-26

Problem 1. Let $f(x,y) = \frac{7}{8}(y^2 + y + x)^4$. Compute $f_{xyxyx}(2,0)$. Hint: It is more convenient to do all the x-derivatives first. (Why are you allowed to do that?)

Problem 2. Find a function $f(x,y)$ such that

$$f(0,0) = 3, f_x(0,0) = -2, f_y(0,0) = 0, f_{xx}(0,0) = 7, f_{xy}(0,0) = -8, f_{yy}(0,0) = 3.$$

(If you don't know where to start, perhaps try solving a single-variable version of the problem first. That is, can you find a function $f(x)$ with $f(0) = 3$, $f'(0) = 2$, and $f''(0) = 4$ for example?)

Problem 3. Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2,1,3)$. You don't have an equation for S but you know that the curves

$$\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle$$
$$\mathbf{r}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle$$

both lie on S. Find an equation of the tangent plane at P.

Problem 4. The intersection of the plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse C. Find the tangent line to C at the point $(-1,-1,2)$.

Problem 5. The cylinder $x^2 + y^2 = 1$ intersects the hyperbolic paraboloid $z = xy$ in a curve C. Find the tangent line to C at the point $(3/5, 4/5, 12/25)$.

Problem 1. By Clairaut's theorem, this is the same as $f_{xxxyy}(2,0) = \boxed{42}$.

Problem 2. The following function will suffice, as you can check:

$$f(x,y) = 3 - 2x + \frac{7}{2}x^2 - 8xy + \frac{3}{2}y^2 + 87x^{30}.$$

Of course the last term is completely unnecessary.

Problem 3. $\mathbf{r}_1(0) = \mathbf{r}_2(1) = \langle 2,1,3\rangle$. $\mathbf{r}_1'(0) = \langle 3,0,-4\rangle$ and $\mathbf{r}_2'(1) = \langle 2,6,2\rangle$. Use their cross product as a normal vector for the plane:

$$24(x-2) - 14(y-1) + 18(z-3) = 0.$$

Problem 4. The tangent plane to the paraboloid at the point in question is

$$z = 2 - 2(x+1) - 2(y+1) = -2 - 2x - 2y.$$

The tangent line to C at the point is the intersection of this plane with the other plane $x + y + 2z = 2$. This can be parametrized as

$$\mathbf{L}(t) = \langle -1,-1,2\rangle + t\langle -3,3,0\rangle.$$

Problem 5. This problem could be solved similarly to the preceding one. Or you could explicitly parametrize C, say as $\mathbf{r}(t) = \langle \cos t, \sin t, \sin t \cos t\rangle$. Then

$$\mathbf{r}'(t) = \langle -\sin t, \cos t, \cos^2 t - \sin^2 t\rangle = \langle -y, x, x^2 - y^2\rangle.$$

So our desired tangent line is

$$\mathbf{L}(t) = \langle 3/5, 4/5, 12/25\rangle + t\langle -4/5, 3/5, -7/25\rangle.$$